

- iv) Consider the following two statements. 02 L₁ CO4
- I) If $\sum a_n$ converges then $\lim_{n \rightarrow \infty} a_n = 0$.
- II) If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum a_n$ converges. Then
- a) both I and II are true, b) only I is true,
c) both I and II are false, d) only II is true.
- v) Consider the following two statements. 02 L₂ CO4
- I) $\sum \frac{1}{\sqrt[3]{n}}$, II) $\sum \frac{1}{n^3 + n}$.
- a) both I and II are convergent, b) only I is convergent,
c) both I and II are not convergent, d) only II is convergent.
- vi) Which one of the following Series is divergent 02 L₁ CO4
- a) $\sum \frac{1}{(2n-1)(2n)}$, b) $\sum \frac{2^n}{n}$, c) $\sum \frac{n^2}{n!}$, d) $\sum \frac{1}{\sqrt[3]{n^5}}$.
- vii) If $S_n = 1 + \left[\frac{(-1)^n}{n} \right]$ then the sequence $\{S_n\}$ is 02 L₁ CO5
- a) converges to 1 and bounded,
b) converges to 0 and bounded,
c) converges to 1 and unbounded,
d) converges to 0 and unbounded.
- viii) $\lim_{n \rightarrow \infty} \frac{x^2 + nx}{n} \forall x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) x.
- ix) $\lim_{n \rightarrow \infty} \frac{nx}{1 + n^2 x^2}$ for all $x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) e.
- x) $\lim_{n \rightarrow \infty} \frac{3x^2 + 7}{n} \forall x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) not exist.

ESE



Sanjay Ghodawat University, Kolhapur

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EXM/P/09/01

Year and Program: 2018-19

School of Science

Department of Mathematics

B.Sc.II

Course Code: MTS 201

Course Title: Mathematics III

Semester – III

Day and Date: Monday
25-11-2018

End Semester Examination
(ESE)

Time: 2.5 Hrs
Max Marks: 100

11.00 to 1.30 PM

Instructions:

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary.
- 3) Figures to the right indicate full marks.

Q.2	Solve any Two of the following.	Marks	Bloom's Level	CO
a)	Prove that every subset of countable set is countable.	06	L ₂	CO1
b)	Prove that any open interval (a, b) is equivalent to any other open interval (c, d) .	06	L ₂	CO1
c)	Show that for each $n \in \mathbb{N}$, the sum of the first n natural numbers is given by $\frac{1}{2}n(n+1)$.	06	L ₂	CO1
Q3				
Solve any Two of the following.				
a)	If x and y are any real numbers with property $x < y$, then prove that there exist a rational number $r \in \mathbb{Q}$ such that $x < r < y$.	07	L ₃	CO2
b)	If $x > -1$ then prove that $(1+x)^n \geq 1+nx$.	07	L ₃	CO2
c)	Determine the set A of $x \in \mathbb{R}$ such that $ 2x+3 < 7$.	07	L ₃	CO2
Q4				
Solve any Two of the following.				
a)	Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n > 0$. Then show that the sequence $\sqrt{x_n}$ of positive square roots converges and $\lim_{n \rightarrow \infty} (\sqrt{x_n}) = \sqrt{x}$.	07	L ₂	CO3
b)	Let $X = (x_n)$ be a bounded sequence of real numbers where $x \in \mathbb{R}$, have the property that every convergent subsequence of X converges to x . Then prove that the sequence X converges to x .	07	L ₂	CO3

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- c) Show that, a monotone sequence of real numbers is convergent if and only if it is bounded. Further if $X = (x_n)$ is bounded increasing sequence, then $\lim_{n \rightarrow \infty} (x_n) = \sup\{x_n : n \in N\}$. 07 L₂ CO3

Q5

- a) **Solve any Three of the following.**
- i) If a series $\sum x_n$ is convergent, then show that any series obtained from it by grouping the terms is also convergent and to the same value. 05 L₃ CO4
- ii) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ is convergent. 05 L₃ CO4
- iii) Establish the convergence or the divergence of the series whose n^{th} term is $\frac{n}{(n+1)(n+2)}$. 05 L₃ CO4
- iv) Establish the convergence or the divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$. 05 L₃ CO4

- b) Establish the convergence or the divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$. 05 L₃ CO4

- Q.6 a) Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f . Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$. 08 L₄ CO5

- b) **Solve any Two of the following.**
- i) A sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \rightarrow 0$. 06 L₄ CO5

- ii) Show that $\lim_{n \rightarrow \infty} \left(\frac{1}{n} \sin(nx + n) \right) = 0$ for $x \in R$. 06 L₄ CO5

- iii) Let (f_n) be a sequence of continuous functions on a set $A \subseteq R$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow R$. Then show that f is continuous on A. 06 L₄ CO5

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