

Year and Program:2019-20

School of Science

Department of Mathematics

B.Sc.II

Course Code: MTS 201

Course Title: Mathematics III

Semester – III

Day and Date: Monday
25-11-2019

End Semester Examination (ESE)

Time: Hrs. 10.30 to 11.00 pm.

Max Marks: 100

PRN/Exam seat No:

Answer booklet No:

Student's signature:

Invigilator signature:

Instructions:

- 1) All questions are compulsory.
- 2) **Attempt Q.1 within first 30 minutes.**
- 3) Each MCQ type question is followed by four plausible alternatives, Tick (✓) the correct one.
- 4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
- 5) If you tick more than one option it will not be evaluated
- 6) Figures to the right indicate full marks
- 7) Use **Blue ball pen** only.

Q.1	Choose the correct alternative for following questions.	Marks	Bloom's Level	CO
i)	Consider the following two statements. I) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$ II) $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$, where A, B, C are sets, then a) only I is true, b) only II is true, c) both I and II are true, d) both I and II are false.	02	L ₁	CO1
ii)	The set A of all real numbers x such that $2x + 3 \leq 6$ then a) $A = \{x \in R : x \leq \frac{3}{2}\}$, b) $A = \{x \in R : x < \frac{3}{2}\}$, c) $A = \{x \in R : x > \frac{3}{2}\}$, d) $A = \{x \in R : x \geq \frac{3}{2}\}$.	02	L ₂	CO2
iii)	Consider the following two statements. I) Every bounded sequence in R contains convergent subsequence. II) A sequence $\{P_n\}$ Converges to p, if every subsequence of $\{P_n\}$ converges to p. Then a) both I and II are true b) only I is true, c) both I and II are false, d) only II is true.	02	L ₁	CO3

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- iv) Consider the following two statements. 02 L₁ CO4
- I) If $\sum a_n$ converges then $\lim_{n \rightarrow \infty} a_n = 0$.
- II) If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum a_n$ converges. Then
- a) both I and II are true, b) only I is true,
c) both I and II are false, d) only II is true.
- v) Consider the following two statements. 02 L₂ CO4
- I) $\sum \frac{1}{\sqrt[3]{n}}$, II) $\sum \frac{1}{n^3 + n}$.
- a) both I and II are convergent, b) only I is convergent,
c) both I and II are not convergent, d) only II is convergent.
- vi) Which one of the following Series is divergent 02 L₁ CO4
- a) $\sum \frac{1}{(2n-1)(2n)}$, b) $\sum \frac{2^n}{n}$, c) $\sum \frac{n^2}{n!}$, d) $\sum \frac{1}{\sqrt[3]{n^5}}$.
- vii) If $S_n = 1 + \left[\frac{(-1)^n}{n} \right]$ then the sequence $\{S_n\}$ is 02 L₁ CO5
- a) converges to 1 and bounded,
b) converges to 0 and bounded,
c) converges to 1 and unbounded,
d) converges to 0 and unbounded.
- viii) $\lim_{n \rightarrow \infty} \frac{x^2 + nx}{n} \quad \forall x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) x.
- ix) $\lim_{n \rightarrow \infty} \frac{nx}{1 + n^2 x^2}$ for all $x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) e.
- x) $\lim_{n \rightarrow \infty} \frac{3x^2 + 7}{n} \quad \forall x \in R$ 02 L₁ CO5
- a) 1, b) 0, c) -1, d) not exist.

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Sanjay Ghodawat University, Kolhapur

2018-19

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EXM/P/09/01

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Department of Mathematics

B.Sc.II

Course Code: MTS 201

Course Title: Mathematics III

Semester – III

Day and Date: Monday
25-11-2018

End Semester Examination
(ESE)

Time: 2.5 Hrs
Max Marks: 100

11:00 to 1:30 pm

Instructions:

- 1) All questions are compulsory.
- 2) Assume suitable data wherever necessary.
- 3) Figures to the right indicate full marks.

Q.2	Solve any Two of the following.	Marks	Bloom's Level	CO
a)	Prove that every subset of countable set is countable.	06	L ₂	CO1
b)	Prove that any open interval (a, b) is equivalent to any other open interval (c, d) .	06	L ₂	CO1
c)	Show that for each $n \in N$, the sum of the first n natural numbers is given by $\frac{1}{2}n(n+1)$.	06	L ₂	CO1
Q3	Solve any Two of the following.			
a)	If x and y are any real numbers with property $x < y$, then prove that there exist a rational number $r \in Q$ such that $x < r < y$.	07	L ₃	CO2
b)	If $x > -1$ then prove that $(1+x)^n \geq 1+nx$.	07	L ₃	CO2
c)	Determine the set A of $x \in R$ such that $ 2x+3 < 7$.	07	L ₃	CO2
Q4	Solve any Two of the following.			
a)	Let $X = (x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n > 0$. Then show that the sequence $\sqrt{x_n}$ of positive square roots converges and $\lim_{n \rightarrow \infty} (\sqrt{x_n}) = \sqrt{x}$.	07	L ₂	CO3
b)	Let $X = (x_n)$ be a bounded sequence of real numbers where $x \in R$, have the property that every convergent subsequence of X converges to x . Then prove that the sequence X converges to x .	07	L ₂	CO3

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	c)	Show that, a monotone sequence of real numbers is convergent if and only if it is bounded. Further if $X = (x_n)$ is bounded increasing sequence, then $\lim_{n \rightarrow \infty} (x_n) = \sup\{x_n : n \in N\}$.	07	L ₂	CO3
Q5	a)	Solve any Three of the following.			
	i)	If a series $\sum x_n$ is convergent, then show that any series obtained from it by grouping the terms is also convergent and to the same value.	05	L ₃	CO4
	ii)	Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ is convergent.	05	L ₃	CO4
	iii)	Establish the convergence or the divergence of the series whose n^{th} term is $\frac{n}{(n+1)(n+2)}$.	05	L ₃	CO4
	iv)	Establish the convergence or the divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.	05	L ₃	CO4
	b)	Establish the convergence or the divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$.	05	L ₃	CO4
Q.6	a)	Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f . Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$.	08	L ₄	CO5
	b)	Solve any Two of the following.			
	i)	A sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\ f_n - f\ _A \rightarrow 0$.	06	L ₄	CO5
	ii)	Show that $\lim_{n \rightarrow \infty} \left(\frac{1}{n} \sin(nx + n) \right) = 0$ for $x \in R$.	06	L ₄	CO5
	iii)	Let (f_n) be a sequence of continuous functions on a set $A \subseteq R$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow R$. Then show that f is continuous on A.	06	L ₄	CO5

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