Year : 2019-20	**School of Science**	**Department of Mathematics**
Program: B.Sc. II		
Course Code – MTS201A	**Course Title – Mathematics III**	**Semester – III**
Day and Date - Monday 25-11-2019	**End Semester Examination**	**Time:** $\frac{1}{2}$ Hr. 10.30 to 11.00 am
		Max Marks: 100
PRN number –	**Seat no-**	**Answer Booklet No.-**
Students' Signature -		**Invigilator's Signature –**

Instructions:

1) All questions are compulsory.
2) **Attempt Q.1 within first 30 minutes.**
3) Each MCQ type question is followed by four plausible alternatives, Tick (√) the correct one.
4) Answer to question 1 should be written in the question paper and submit to the Jr. Supervisor.
5) If you tick more than one option it will not be evaluated
6) Figures to the right indicate full marks
7) Use **Blue ball pen** only.

Q.1	Tick mark (√) the correct alternative	Marks	Bloom's Level	COs
i)	Let A and B be Sets such that A is countable and B is uncountable then $A \cap B$ is a) countable , b) uncountable , c) finite, d) none of these.	02	L1	CO 1
ii)	Consider the following two statements I) If x and y are real then $\|\|x\|-\|y\|\| = \|x-y\|$. II) If $x \in R^k$ and $y \in R^k$ then $\|x-y\|^2 + \|x+y\|^2 = 2\|x\|^2 + 2\|y\|^2$. Then a) both I & II are true , b) only II is true , c) only I is true , d) both I & II are false .	02	L2	CO 2
iii)	Consider the following two statements I) Every Cauchy sequence is convergent. II) Every convergent sequence need not be Cauchy. a) both I & II are true , b) only II is true , c) only I is true , d) both I & II are false .	02	L1	CO 3

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iv) Consider the following two statements — 02 — L1 — CO 4

I) If $\sum_{n=1}^{\infty} a_n$ converges then $\lim_{n\to\infty} a_n = 0$

II) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.

Then

a) only I is true , b) only II is true ,
c) both I & II are true , d) both I & II are false .

v) If the series $\sum_{n=1}^{\infty} x_n$ converges then $\lim_{n\to\infty}(x_n)$ is equal to — 02 — L2 — CO 4

a) 0 , b) 1 , c) −1 , d) 1/2 .

vi) The series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ converges to — 02 — L1 — CO 4

a) 1/6 , b) 1 , c) 1/4 , d) 1/2 .

vii) Which one of the following series is convergent series? — 02 — L1 — CO 4

a) $\sum_{n=1}^{\infty} \frac{1}{n}\cos\left(\frac{1}{n}\right)$, b) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$,

c) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$, d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[7]{n^5}}$.

viii) $\lim_{n\to\infty} \frac{1}{n^4}\sum_{j=1}^{n} j^3$ is equal to — 02 — L1 — CO 5

a) 1/4 , b) 1 , c) 1/5 , d) 1/2 .

ix) Consider the following two statements — 02 — L1 — CO 5

I) Let $f \in R[a,b]$ then f is bounded on $[a,b]$.

II) Every step function $\phi : [a,b] \to \mathbb{R}$ is Riemann integrable, $\phi \in R[a,b]$,

Then

a) only A is true , b) only B is true ,
c) both A & B are true , d) both A & B are false.

x) If $f(x) = x^2$; $x \in [0,4]$ then the Riemann sum $S(f;\dot{P})$ where $\dot{P} = (0,1,2,4)$ with tag at left end points of the subintervals is — 02 — L1 — CO 5

a) 37 , b) 10 , c) 1/9 , d) 9 .

ESE

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EXM/P/09/01

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Program:B.Sc. II

Course Code: MTS201A | **Course Title: Mathematics III** | **Semester – III**

Day and Date: Monday 25-11-2019 | **End Semester Examination (ESE)** | **Time:** 2.5 Hr. 11.00 to 1.30

Max Marks: 100

Instructions:
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Non-programmable calculator is allowed

			Marks	Bloom's Level	COs
Q.2		Solve any **TWO** of the following.			
	i)	If the function $f : A \to B$ has an inverse then show that the function f is one-one and onto.	06	L3	CO1
	ii)	Prove that the set of all rational numbers is countable.	06	L2	CO1
	iii)	Prove that $2^n \leq (n+1)!$ holds for all $n \epsilon \mathbb{N}$.	06	L2	CO1
Q.3		Solve any **TWO** of the following.			
	i)	If $x \epsilon \mathbb{R}$, then prove that there exists $n_x \epsilon \mathbb{N}$ such that $x < n_x$.	07	L2	CO2
	ii)	State and prove the Bernoulli's inequality.	07	L2	CO2
	iii)	Is there exist a rational number r such that $r^2 = 2$? Justify your answer.	07	L3	CO2
Q.4		Solve any **TWO** of the following.			
	i)	If $X = \{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers, then show that there is a subsequence of X that is monotone.	07	L3	CO3
	ii)	State and prove the Bolzano Weierstrass theorem. Give an example of bounded sequence which is not convergent.	07	L2	CO3
	iii)	Let $X = \{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers that converges to x and suppose that $x_n > 0$, then prove that the sequence $\left\{\sqrt{x_n}\right\}_{n=1}^{\infty}$ of positive square roots is converge and $\lim_{n\to\infty}\left(\sqrt{x_n}\right) = \sqrt{x}$.	07	L2	CO3
Q.5	i)	Test the convergence of the series $\frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \ldots, x > 0$.	06	L3	CO4

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	ii)	Solve any **TWO** of the following.			
		a) Discuss the convergence of the geometric series given by $\sum_{n=1}^{\infty} r^n$.	07	L2	CO4
		b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent while $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.	07	L3	CO4
		c) Test the convergence of the series $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3})$.	07	L3	CO4
Q.6	i)	Let $h(x) = x$ for $x \in [0,1]$, then show that $h \in R[0,1]$.	08	L4	CO5
	ii)	Solve any **TWO** of the following.			
		a) Let $f \in R[a,b]$ and if α, β, γ are any numbers in $[a,b]$ then show that $\int_{\alpha}^{\beta} f = \int_{\alpha}^{\gamma} f + \int_{\gamma}^{\beta} f$.	06	L3	CO5
		b) Find the Riemann integral of $\int_{1}^{4} \frac{\sin\sqrt{t}}{\sqrt{t}}\, dt$.	06	L3	CO5
		c) Prove that Every continuous function on $[a,b]$ is in $R[a,b]$.	06	L4	CO5

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